NAG Toolbox for MATLAB

f07th

1   Purpose

f07th returns error bounds for the solution of a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^{\mathrm{T}}X = B$.

2   Syntax

```
[ferr, berr, info] = f07th(uplo, trans, diag, a, b, x, 'n', n, 'nrhs_p',
nrhs_p)
```

3   Description

f07th returns the backward errors and estimated bounds on the forward errors for the solution of a real triangular system of linear equations with multiple right-hand sides $AX = B$ or $A^{\mathrm{T}}X = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of f07th in terms of a single right-hand side b and solution x.

Given a computed solution x, the function computes the *component-wise backward error* β. This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$
$$\left|\delta a_{ij}\right| \leq \beta\left|a_{ij}\right| \qquad \text{and} \qquad |\delta b_i| \leq \beta|b_i|.$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i|x_i - \hat{x}_i| / \max_i|x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the F07 Chapter Introduction.

4   References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5   Parameters

5.1   Compulsory Input Parameters

1:    **uplo – string**

Indicates whether A is upper or lower triangular.

uplo $=$ 'U'

   A is upper triangular.

uplo $=$ 'L'

   A is lower triangular.

Constraint: **uplo** $=$ 'U' or 'L'.

2:    **trans – string**

Indicates the form of the equations.

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T' or 'C'

The equations are of the form $A^{\mathrm{T}}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3:    **diag – string**

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4:    **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular matrix A.

If **uplo** = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5:    **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

6:    **x(ldx,∗) – double array**

The first dimension of the array **x** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X, as returned by f07te.

5.2   Optional Input Parameters

1:    **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the arrays **b**, **x**. (An error is raised if these dimensions are not equal.)

r, the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldx, work, iwork

5.4 Output Parameters

1: **ferr**$(*)$ **– double array**

Note: the dimension of the array **ferr** must be at least $\max(1, \text{nrhs\_p})$.

ferr(j) contains an estimated error bound for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

2: **berr**$(*)$ **– double array**

Note: the dimension of the array **berr** must be at least $\max(1, \text{nrhs\_p})$.

berr(j) contains the component-wise backward error bound β for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **nrhs_p**, 6: **a**, 7: **lda**, 8: **b**, 9: **ldb**, 10: **x**, 11: **ldx**, 12: **ferr**, 13: **berr**, 14: **work**, 15: **iwork**, 16: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

A call to f07th, for each right-hand side, involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{T}x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately n^2 floating-point operations.

The complex analogue of this function is f07tv.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
a = [4.3, 0, 0, 0;
     -3.96, -4.87, 0, 0;
     0.4, 0.31, -8.02, 0;
     -0.27, 0.07, -5.95, 0.12];
b = [-12.9, -21.5;
     16.75, 14.93;
     -17.55, 6.33;
     -11.04, 8.09];
[x, info] = f07te(uplo, trans, diag, a, b);
[ferr, berr, info] = f07th(uplo, trans, diag, a, b, x)

ferr =
   1.0e-13 *
   0.8331
   0.2637
berr =
   1.0e-16 *
   0.6885
        0
info =
        0
```